I. Experimental test of non-macrorealistic cat states in the cloud by Huan-Yu Ku

Definition 1.1. Any macrorealistic system fulfill the **Leggett-Garg ineqaulity** which is a result of the following two postulates: 1.) macrorealism and 2.) Noninvasive measurement. The former says that the a macroscopic system is in some definite state at any given time, and the latter states that some measurement can be performed to preserve the original state.

If a system exists in a definite state at any times, and the measurement is non-invasive, the probability of any outcomes, j, at times t_2 can then be written as the following equation, regardless of the system begin measured at time t_1 or not:

$$P_{t_2}(j) = P_{t_2}^M(j) = \sum_i P_{t_1}(i)P_{t_2}(j|i) \tag{1}$$

where the "invasiveness witness" is written as $W = |P_{t_2}(j) - P_{t_2}^M(j)|$, where W is 0 when the measurement is non-invasive

The noninvasive measurement is not realistic any may require some minor adjustment such that the state is changed when it's measured in time t_1, the the equation (1) is modified to:

$$P_{t_2}(j) = P_{t_2}^M(j) = \sum_{i,k} \epsilon_{t_1}^M P_{t_1}(i)P_{t_2}(j|i) \tag{2}$$

then the "invasiveness witness" can be changed from the one with true non-invasiveness to the one with "clumsy non-invasiveness":

$$W = \max[|1 - \epsilon_{t_1}^M(i|i)|] \tag{3}$$

where it can be seen that if $\epsilon_{t_1}^M = 1$, representing that the existence of measurement does not change the weight of the outcomes probability at time t_1, the outcomes probability changed to equation (1), where if the $\epsilon_{t_1}^M = 0$, the system is so strongly disturbed the state changed into some state that is orthogonal to the original state.

To quantify if the system is of macro-realistic or of quantum characteristic, the "disconnetivity" of a system is used.

Definition 1.2. disconnetivity is defined as the number of correlations between each "branch of superposition" one need to measure to distinguish the otherwise two indistinguishable states. The disconnectivity,

Γ can then be described as the following:

$$\Gamma = \max_i n_i \quad \ni \frac{S_{n_i}}{min_m(S_m + S_{n_i - m})} < \eta \tag{4}$$

Where S is the von-Neumann entropy and η is the some bound between classical mixtures and entangled states.($\eta = 0.5$ is suggested for some reason unsepecified.)

To initialize a state with the most dsconnectivity(which is highly entangled), a cat state is generated:

Definition 1.3. A **cat state** is refered to a state that is in superposition of all-spin-up or all-spin-down states(GHZ state is one of them), the state can be generally written as:

$$|\phi\rangle_{t_1} = \cos\frac{\theta}{2}|0\rangle^{\otimes n} + \sin\frac{\theta}{2}|1\rangle^{\otimes n} \tag{5}$$

Assuming a non-invasive measurement is performed by a unitary and its conjugate. The "invasiveness witness" can be written as:

$$W = |P_{t_2}(j) - P^M_{t_2}(j)| = |P_{t_1}(i)P_{t_2}(j|i) - P_{t_1}(i)P^M_{t_2}(j|i)| = 1 - \cos^4\frac{\theta}{2} - \sin^4\frac{\theta}{2} \tag{6}$$

As a comparison, assume a state with no entanglement:

$$|\psi\rangle = \frac{1}{\sqrt{2^n}}(|0\rangle + |1\rangle)^{\otimes n} \tag{7}$$

The "invasiveness witness" is written as:

$$W = 1 - \frac{1}{2^n} \tag{8}$$

NOTE The "invasiveness witness" in equation (8) can be acted as a dimensionality witness since the effective dimension of a cat state is very low since it's only dominated by two state: all-spin-up or all-spin-down states.